Windham Capital Management, LLC

CAPM Investors Do Not Get Paid For Bearing Risk (Markowitz)

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Summary

- Investors trade until the marginal utilities of all securities are the same.
- Therefore, expected returns, in part, determine the weights of the market portfolio.
- The market portfolio, in part, determines each securities beta.
- It thus follows that expected returns determine beta and not the other way around.

Example

- Suppose the risk-free rate is zero.
- Two uncorrelated stocks of equal variance, $V_1 = V_2$
- Return of stock 1, $e_1 = 10\%$
- Return of stock 2, $e_2 = 5\%$

Compute the market portfolio

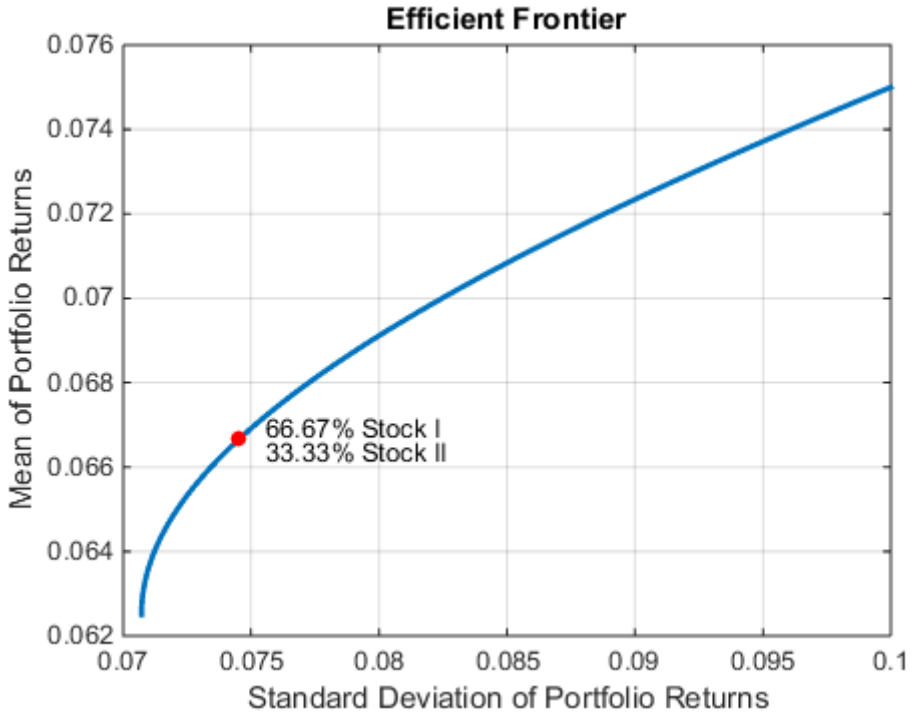
- Mean-variance utility function

$$\mathrm{U}(X) = E[r]'X - \frac{\lambda}{2}X'VX$$

- Subject to $\sum_{i=1}^{N} X_i = 1$ and $0 \leq X_i \leq 1$
- Set the partial derivatives to zero, $\delta_X U = 0$
- The optimal portfolio allocation is

$$X_1 = 66.67\%$$

$$X_2 = 33.33\%$$



$$X_i = \lambda(e_i/V_i)$$

Calculate beta

- Regress security 1 and security 2 against the efficient market portfolio

$$\beta = \frac{VX}{X'VX}$$

- Beta of each security is

$$\beta_1 = 1.2$$

$$\beta_2 = 0.6$$

Now suppose

- Return of stock 1, $e_1 = 5\%$
- Return of stock 2, $e_2 = 10\%$
- The optimal portfolio allocation is now

$$X_1 = 33.33\%$$

$$X_2 = 66.67\%$$

Recalculate Beta

- Beta of each security is

$$\beta_1 = 0.6$$

$$\beta_2 = 1.2$$

- Recall that $X_i = \lambda(e_i/V_i)$

- $V_1 = V_2, e_1 = \frac{1}{2}e_2 \rightarrow \beta_1 = \frac{1}{2}\beta_2$

Conclusion

- CAPM holds not because investors get paid for bearing risk, but
- Investors are induced to invest in security 1 to the point where the ratio of margin effect for security 1 and security 2 are the same.

Practical Implications

- Can extend to correlated case.
- Does not invalidate CAPM, alternative interpretation.
- Estimate beta for new securities.
- Estimate expected return assuming incomplete knowledge of market components.

References

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